GPGN 306 Linear Systems

Catalog Description: Beginning with simple linear systems of coupled elements (springs and masses or electrical circuits, for instance) we study linearity, superposition, damping, resonance and normal modes. As the number of elements increases we end up with the wave equation, which leads, via separation of variables, to the first signs of Fourier series. One of the unifying mathematical themes in this course is orthogonal decomposition, which we first encounter in the comfort of finite dimensional vector spaces associated with springs and masses. But the idea extends naturally to infinite dimensional spaces where it appears as a Fourier series. The course culminates in an exposition of Fourier series, integrals and transforms, both discrete and continuous. Throughout, these ideas are motivated by and applied to current geophysical problems such as normal mode seismology, acoustic wave propagation and spectral analysis of time series. In addition to the lectures, there will be classroom and laboratory demonstrations, and all students will complete a variety of computer exercises, using packages such as Mathematica and Matlab.

Text *Linear Systems* by J.A. Scales: freely available on the WWW.

Delivery Mode: Classroom lecture and demonstration, computer exercises

Requirements: Bi-weekly one-hour quizzes. 2 computer exercises per semester. Homework not graded. No mid-term or final, all quizzes and the computer exercises count equally. Some outside readings from journal articles.

- Part 1: Simple Harmonic Motion: From Springs to Waves
  - A Spring and a mass
  - Simple harmonic oscillation
  - Spring/mass or spring + mass?
  - Sine or cosine? The origin of time.
  - Energy is conserved
  - Forced motion
  - Forced and free oscillations
  - Normal modes

  - Complex numbers and constant coefficient differential equations
  - Forced motion with damping
  - Damped transient motion
  - Another velocity-dependent force: the Zeeman effect and degeneracy
  - Coupled masses
  - Coupled LRC circuits
  - A Matrix appears
  - Matrices for two degrees of freedom
  - The energy method: Hamiltonian systems
  - Matrices for n-dimensional coupled linear systems

- Part 2: Waves and Modes in One and Two Spatial Dimensions

  - 1-D Separation of variables–Fourier Series
  - 2-D separation of variables
  - Normal modes
  - Degeneracy
  - Pictures of measured modes and resonances from many fields

- Part 3: A Little Linear Algebra

  - Linear Vector Spaces
  - Matrices
  - Matrix and vector norms
  - Projecting vectors onto other vectors
  - Subspaces, dimension, span and bases
  - Linear dependence and independence
  - The four fundamental spaces associated with a matrix
  - Gaussian elimination
  - Matrix Inverses, left, right and left and right

- Eigenvalues and eigenvectors
- Orthogonal decomposition of rectangular matrices
- Eigenvectors and orthogonal projections
- Linear response theory
- Green's functions and orthogonal expansions
- SVD and least squares by example
- A worked example: the generalized inverse
    1. No null space
    2. A data null space
    3. A geometrical interpretation of least squares
    4. A model null space
    5. Both a model and a data null space

- Part 4: Fourier Analysis
    - Motivation
    - The Fourier Series on finite intervals
    - Superposition and orthogonal projection
    - The Fourier Series on an infinite interval
    - The Fourier Integral
    - Normalization
    - Invertibility: the Dirichlet Kernel
    - Some Basic Theorems for the Fourier Transform
        1. convolution
        2. shift
        3. scaling
    - Linear filters: design and algebraic interpretation
    - The Discrete Fourier Transform
    - The Linear Algebra of the DFT
    - The DFT from the Fourier Integral

- Discrete Fourier Transform examples
- Convergence theorems
- Basic Properties of Delta functions
- Operational calculus and distributions